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Calculation of the Radar Cross Section of a Perfectly Conducting Sphere

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13. ABSTRACT

A Fortran computer program has been written to compute the radar cross section of a conducting sphere. The program is useful when a metal sphere is used as a standard target for calibrating a radar for target-cross-section measurement. It has been used to machine-plot with high precision a curve of cross section (normalized to the optical cross section) as a function of the radius/wavelength ratio. The computed data used to plot the curve are also presented in tabular form. The mathematics of the problem are briefly reviewed, and a listing of the computed program is given.

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CONTENTS

ABSTRACT

A Fortran computer program has been written to compute the radar cross section of a conducting sphere. The program is useful when a metal sphere is used as a standard target for calibrating a radar for target-cross-section measurement. It has been used to machine-plot with high precision a curve of cross section (normalized to the optical cross section) as a function of the radius/wavelength ratio. The computed data used to plot the curve are also presented in tabular form. The mathematics of the problem are briefly reviewed, and a listing of the computer program is given.

AUTHORIZATION

NRL Problem R02-55
Project RF-151-402-4011

CALCULATION OF THE RADAR CROSS SECTION OF A PERFECTLY CONDUCTING SPHERE

INTRODUCTION

This calculation is of interest because spheres of high-conductivity material (metal) are used as standard targets for calibrating a radar for target cross-section measurement. Published curves based on calculations are unsatisfactory for precise work because they appear to be imprecisely plotted (1) or the scale size is too small for accurate reading of values (2,3). Therefore the problem of writing a computer program to calculate the cross section and to plot it accurately by machine was undertaken.

The computation requires summing a series whose terms involve spherical Bessel and Hankel functions. These terms are complex quantities. Manual calculation would be extremely laborious and computer calculation also poses some difficult problems. After a successful program had been written, it was learned that others had also done it (3). Their programming approach was somewhat different, and their actual program is not presented in their paper. The program developed here is presented in a form that allows it to be used for the calculation of cross sections of specific sphere sizes, by reading in data giving the radius of the sphere and the radar frequency. This procedure would be used if the curve and table do not provide a sufficiently accurate number.

RESULTS

The computer program was used to machine plot a curve, Fig. 1, of the normalized radar cross section $\sigma/\pi a^2$, where σ is the actual cross section and a is the radius of the sphere, as a function of the ratio a/λ, where λ is the wavelength. The normalizing quantity πa^2 is of course the optical cross section of the sphere. The plot was made in a large size (20 x 25 inches) on the NRL Gerber Model 875 Automatic Drafting Machine, with an accuracy of the order of .001 inch. The coordinate grid was plotted by the machine also, so that there are no registration errors, as might occur if the plot were made on standard graph paper.

The computations were made for values of a/λ from .05 to 5.045 in steps of .005. These results are tabulated in Table 1. The plot was also made from these values. Even though the a/λ interval .005 is quite small, a direct digital plot does not result in a perfectly smooth curve. Therefore a "smoothing" subroutine, described in a previous NRL report (4), was used for

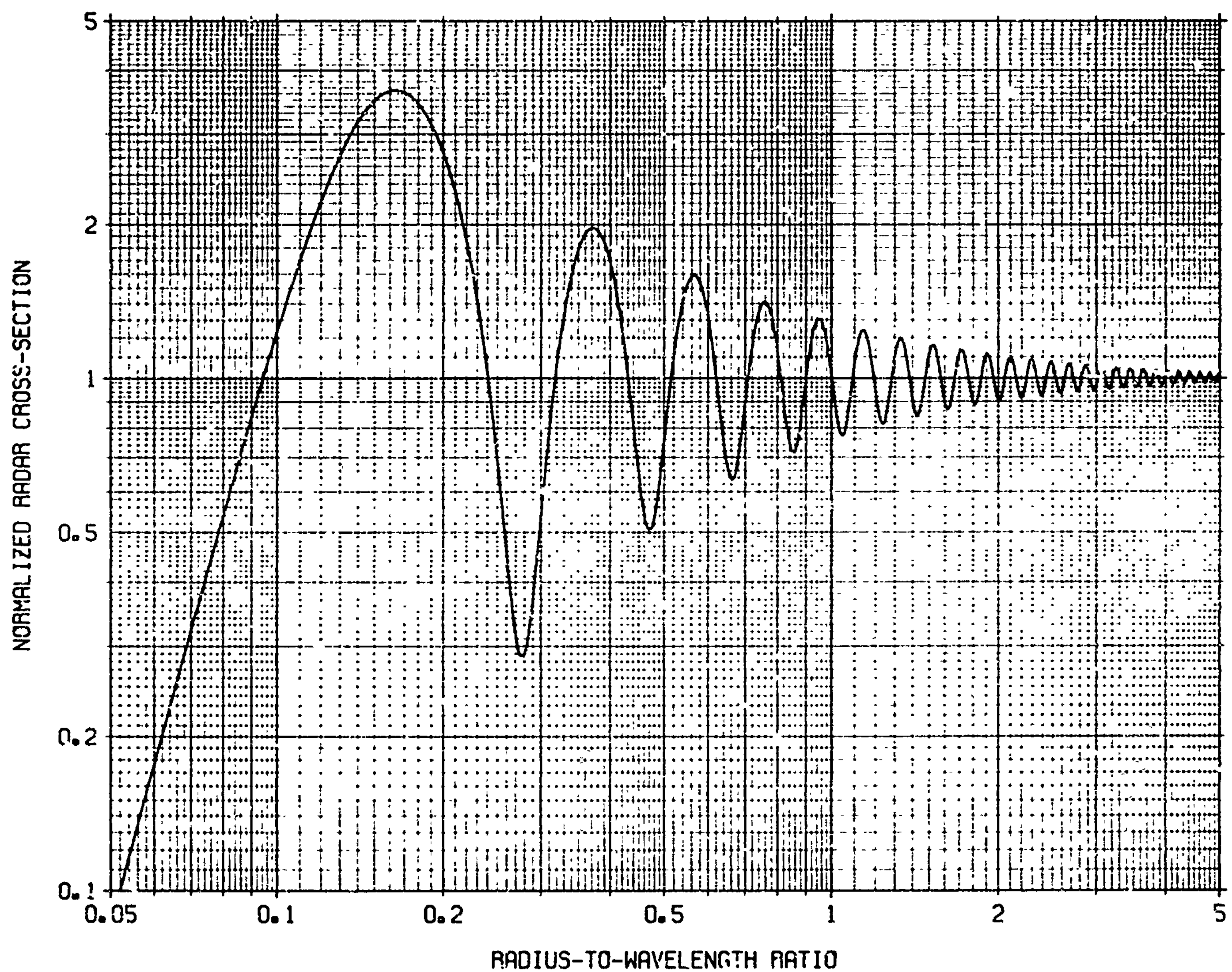

Fig. 1 - Radar cross section of a perfectly conducting sphere, normalized to the optical cross section πa^2, as a function of the radius-to-wavelength ratio a/λ.

Table 1

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
0.0500	0.0860	0.3000	0.5412	0.5500	1.5211	0.8000	1.1222
0.0550	0.1254	0.3050	0.6544	0.5550	1.5567	0.8050	1.0659
0.0600	0.1767	0.3100	0.7806	0.5600	1.5793	0.8100	1.0096
0.0650	0.2418	0.3150	0.9158	0.5650	1.5884	0.8150	0.9550
0.0700	0.3229	0.3200	1.0557	0.5700	1.5840	0.8200	0.9034
0.0750	0.4218	0.3250	1.1963	0.5750	1.5663	0.8250	0.8551
0.0800	0.5404	0.3300	1.3337	0.5800	1.5360	0.8300	0.8143
0.0850	0.6800	0.3350	1.4641	0.5850	1.4941	0.8350	0.7790
0.0900	0.8417	0.3400	1.5845	0.5900	1.4417	0.8400	0.7510
0.0950	1.0258	0.3450	1.6919	0.5950	1.3805	0.8450	0.7310
0.1000	1.2315	0.3500	1.7839	0.6000	1.3122	0.8500	0.7194
0.1050	1.4572	0.3550	1.8587	0.6050	1.2388	0.8550	0.7163
0.1100	1.6999	0.3600	1.9150	0.6100	1.1622	0.8600	0.7218
0.1150	1.9549	0.3650	1.9517	0.6150	1.0845	0.8650	0.7355
0.1200	2.2164	0.3700	1.9684	0.6200	1.0080	0.8700	0.7569
0.1250	2.4772	0.3750	1.9653	0.6250	0.9344	0.8750	0.7855
0.1300	2.7294	0.3800	1.9427	0.6300	0.8658	0.8800	0.8203
0.1350	2.9646	0.3850	1.9015	0.6350	0.8037	0.8850	0.8603
0.1400	3.1747	0.3900	1.8431	0.6400	0.7498	0.8900	0.9043
0.1450	3.3524	0.3950	1.7691	0.6450	0.7053	0.8950	0.9512
0.1500	3.4921	0.4000	1.6815	0.6500	0.6711	0.9000	0.9996
0.1550	3.5898	0.4050	1.5827	0.6550	0.6480	0.9050	1.0482
0.1600	3.6435	0.4100	1.4753	0.6600	0.6362	0.9100	1.0957
0.1650	3.6533	0.4150	1.3620	0.6650	0.6359	0.9150	1.1407
0.1700	3.6208	0.4200	1.2458	0.6700	0.6468	0.9200	1.1822
0.1750	3.5490	0.4250	1.1296	0.6750	0.6685	0.9250	1.2191
0.1800	3.4418	0.4300	1.0165	0.6800	0.7001	0.9300	1.250[illegible]
0.1850	3.3038	0.4350	0.9092	0.6850	0.7407	0.9350	1.2753
0.1900	3.1397	0.4400	0.8106	0.6900	0.7889	0.9400	1.2932
0.1950	2.9542	0.4450	0.7228	0.6950	0.8435	0.9450	1.3039
0.2000	2.7522	0.4500	0.6481	0.7000	0.9028	0.9500	1.3069
0.2050	2.5380	0.4550	0.5881	0.7050	0.9653	0.9550	1.3025
0.2100	2.3160	0.4600	0.5439	0.7100	1.0290	0.9600	1.2908
0.2150	2.0900	0.4650	0.5164	0.7150	1.0925	0.9650	1.2721
0.2200	1.8638	0.4700	0.5057	0.7200	1.1539	0.9700	1.2472
0.2250	1.6412	0.4750	0.5118	0.7250	1.2116	0.9750	1.2166
0.2300	1.4255	0.4800	0.5339	0.7300	1.2641	0.9800	1.1813
0.2350	1.2202	0.4850	0.5710	0.7350	1.3101	0.9850	1.1423
0.2400	1.0284	0.4900	0.6217	0.7400	1.3483	0.9900	1.1007
0.2450	0.8533	0.4950	0.6841	0.7450	1.3780	0.9950	1.0575
0.2500	0.6976	0.5000	0.7564	0.7500	1.3983	1.0000	1.0140
0.2550	0.5641	0.5050	0.8363	0.7550	1.4089	1.0050	0.9712
0.2600	0.4548	0.5100	0.9216	0.7600	1.4095	1.0100	0.9304
0.2650	0.3718	0.5150	1.0098	0.7650	1.4004	1.0150	0.8925
0.2700	0.3161	0.5200	1.0986	0.7700	1.3819	1.0200	0.8586
0.2750	0.2884	0.5250	1.1856	0.7750	1.3545	1.0250	0.8294
0.2800	0.2886	0.5300	1.2686	0.7800	1.3193	1.0300	0.8057
0.2850	0.3159	0.5350	1.3454	0.7850	1.2771	1.0350	0.7881
0.2900	0.3688	0.5400	1.4142	0.7900	1.2293	1.0400	0.7770
0.2950	0.4449	0.5450	1.4732	0.7950	1.1772	1.0450	0.7725

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
1.0500	0.7148	1.3000	1.0951	1.5500	1.1124	1.8000	0.8977
1.0550	[illegible]	1.3050	1.1209	1.5550	1.0934	1.8050	0.8911
1.0600	[illegible]	1.3100	1.1436	1.5600	1.0723	1.8100	0.8876
1.0650	[illegible]	1.3150	1.1626	1.5650	1.0496	1.8150	0.8873
1.0700	0.8435	1.3200	1.1774	1.5700	1.0260	1.8200	0.8901
1.0750	0.8758	1.3250	1.1876	1.5750	1.0022	1.8250	0.8959
1.0800	0.9096	1.3300	1.1931	1.5800	0.9787	1.8300	0.9047
1.0850	0.9460	1.3350	1.1936	1.5850	0.9562	1.8350	0.9160
1.0900	0.9840	1.3400	1.1893	1.5900	0.9353	1.8400	0.9297
1.0950	1.0224	1.3450	1.1803	1.5950	0.9165	1.8450	0.9452
1.1000	1.0603	1.3500	1.1669	1.6000	0.9003	1.8500	0.9622
1.1050	1.0966	1.3550	1.1495	1.6050	0.8871	1.8550	0.9802
1.1100	1.1305	1.3600	1.1286	1.6100	0.8772	1.8600	0.9988
1.1150	[illegible]	1.3650	1.1047	1.6150	0.8709	1.8650	1.0173
1.1200	1.1873	1.3700	1.0786	1.6200	0.8682	1.8700	1.0354
1.1250	1.2088	1.3750	1.0510	1.6250	0.8693	1.8750	1.0525
1.1300	1.2250	[illegible]	1.0225	1.6300	0.8741	1.8800	1.0683
1.1350	1.2355	1.3850	0.9940	1.6350	0.8823	1.8850	1.0822
1.1400	1.2400	1.3900	0.9663	1.6400	0.8938	1.8900	1.0940
[illegible]	[illegible]	1.3950	0.9400	1.6450	0.9082	1.8950	1.1032
[illegible]	1.2313	1.4000	0.9158	1.6500	0.9252	1.9000	1.1099
[illegible]	1.2183	1.4050	0.8944	1.6550	0.9442	1.9050	1.1136
[illegible]	1.2001	1.4100	0.8763	1.6600	0.9646	1.9100	1.1145
1.1650	1.1773	1.4150	0.8620	1.6650	0.9861	1.9150	1.1124
1.1700	1.1503	1.4200	0.8517	1.6700	1.0079	1.9200	1.1075
1.1750	1.1201	1.4250	0.8458	1.6750	1.0295	1.9250	1.0998
1.1800	1.0875	1.4300	0.8443	1.6800	1.0503	1.9300	1.0897
1.1850	1.0533	1.4350	0.8473	1.6850	1.0697	1.9350	1.0774
1.1900	1.0184	1.4400	0.8545	1.6900	1.0873	1.9400	1.0633
1.1950	0.9839	1.4450	0.8659	1.6950	1.1027	1.9450	1.0477
1.2000	0.9506	1.4500	0.8809	1.7000	1.1153	1.9500	1.0311
1.2050	0.9193	1.4550	0.8993	1.7050	1.1249	1.9550	1.0140
1.2100	0.8910	1.4600	0.9204	1.7100	1.1313	1.9600	0.9966
1.2150	0.8663	1.4650	0.9437	1.7150	1.1343	1.9650	0.9797
1.2200	0.8458	1.4700	0.9685	1.7200	1.1338	1.9700	0.9635
1.2250	0.8301	1.4750	0.9942	1.7250	1.1300	1.9750	0.9485
1.2300	0.8194	1.4800	1.0200	1.7300	1.1229	1.9800	0.9352
1.2350	0.8141	1.4850	1.0454	1.7350	1.1128	1.9850	0.9237
1.2400	0.8141	1.4900	1.0695	1.7400	1.0999	1.9900	0.9145
1.2450	0.8196	1.4950	1.0918	1.7450	1.0846	1.9950	0.9078
1.2500	0.8301	1.5000	1.1117	1.7500	1.0674	2.0000	0.9037
1.2550	0.8455	1.5050	1.1286	1.7550	1.0486	2.0050	0.9022
1.2600	0.8652	1.5100	1.1423	1.7600	1.0289	2.0100	0.9035
1.2650	0.8887	1.5150	1.1522	1.7650	1.0087	2.0150	0.9075
1.2700	0.9152	1.5200	1.1582	1.7700	0.9887	2.0200	0.9140
1.2750	0.9441	1.5250	1.1602	1.7750	0.9692	2.0250	0.9229
1.2800	0.9745	1.5300	1.1581	1.7800	0.9509	2.0300	0.9338
1.2850	1.0056	1.5350	1.1520	1.7850	0.9342	2.0350	0.9466
1.2900	1.0367	1.5400	1.1422	1.7900	0.9195	2.0400	0.9607
1.2950	1.0667	1.5450	1.1269	1.7950	0.9072	2.0450	0.9760

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
2.0500	0.9918	2.3000	1.0838	2.5500	0.9681	2.8000	0.9609
2.0550	1.0079	2.3050	1.0798	2.5550	0.9588	2.8050	0.9687
2.0600	1.0237	2.3100	1.0739	2.5600	0.9507	2.8100	0.9772
2.0650	1.0389	2.3150	1.0660	2.5650	0.9440	2.8150	0.9864
2.0700	1.0530	2.3200	1.0566	2.5700	0.9389	2.8200	0.9959
2.0750	1.0657	2.3250	1.0458	2.5750	0.9355	2.8250	1.0055
2.0800	1.0767	2.3300	1.0339	2.5800	0.9339	2.8300	1.0149
2.0850	1.0857	2.3350	1.0213	2.5850	0.9341	2.8350	1.0240
2.0900	1.0924	2.3400	1.0082	2.5900	0.9361	2.8400	1.0324
2.0950	1.0968	2.3450	0.9951	2.5950	0.9398	2.8450	1.0399
2.1000	1.0986	2.3500	0.9824	2.6000	0.9452	2.8500	1.0464
2.1050	1.0979	2.3550	0.9702	2.6050	0.9521	2.8550	1.0516
2.1100	1.0947	2.3600	0.9590	2.6100	0.9602	2.8600	1.0555
2.1150	1.0890	2.3650	0.9490	2.6150	0.9694	2.8650	1.0579
2.1200	1.0812	2.3700	0.9405	2.6200	0.9794	2.8700	1.0589
2.1250	1.0714	2.3750	0.9337	2.6250	0.9900	2.8750	1.0583
2.1300	1.0598	2.3800	0.9286	2.6300	1.0008	2.8800	1.0562
2.1350	1.0468	2.3850	0.9259	2.6350	1.0115	2.8850	1.0527
2.1400	1.0328	2.3900	0.9251	2.6400	1.0220	2.8900	1.0478
2.1450	1.0181	2.3950	0.9263	2.6450	1.0318	2.8950	1.0417
2.1500	1.0031	2.4000	0.9295	2.6500	1.0408	2.9000	1.0347
2.1550	0.9882	2.4050	0.9347	2.6550	1.0487	2.9050	1.0267
2.1600	0.9738	2.4100	0.9416	2.6600	1.0553	2.9100	1.0182
2.1650	0.9604	2.4150	0.9501	2.6650	1.0605	2.9150	1.0093
2.1700	0.9481	2.4200	0.9599	2.6700	1.0641	2.9200	1.0002
2.1750	0.9374	2.4250	0.9706	2.6750	1.0660	2.9250	0.9912
2.1800	0.9286	2.4300	0.9825	2.6800	1.0662	2.9300	0.9825
2.1850	0.9218	2.4350	0.9946	2.6850	1.0647	2.9350	0.9744
2.1900	0.9172	2.4400	1.0068	2.6900	1.0615	2.9400	0.9670
2.1950	0.9150	2.4450	1.0189	2.6950	1.0568	2.9450	0.9606
2.2000	0.9151	2.4500	1.0304	2.7000	1.0506	2.9500	0.9554
2.2050	0.9175	2.4550	1.0411	2.7050	1.0432	2.9550	0.9514
2.2100	0.9222	2.4600	1.0508	2.7100	1.0348	2.9600	0.9487
2.2150	0.9290	2.4650	1.0590	2.7150	1.0255	2.9650	0.9474
2.2200	0.9378	2.4700	1.0658	2.7200	1.0156	2.9700	0.9476
2.2250	0.9482	2.4750	1.0708	2.7250	1.0055	2.9750	0.9492
2.2300	0.9600	2.4800	1.0739	2.7300	0.9953	2.9800	0.9522
2.2350	0.9729	2.4850	1.0751	2.7350	0.9853	2.9850	0.9565
2.2400	0.9865	2.4900	1.0744	2.7400	0.9759	2.9900	0.9619
2.2450	1.0004	2.4950	1.0718	2.7450	0.9672	2.9950	0.9684
2.2500	1.0143	2.5000	1.0674	2.7500	0.9594	3.0000	0.9757
2.2550	1.0278	2.5050	1.0612	2.7550	0.9529	3.0050	0.9836
2.2600	1.0406	2.5100	1.0535	2.7600	0.9476	3.0100	0.9920
2.2650	1.0523	2.5150	1.0446	2.7650	0.9439	3.0150	1.0005
2.2700	1.0626	2.5200	1.0345	2.7700	0.9417	3.0200	1.0090
2.2750	1.0712	2.5250	1.0237	2.7750	0.9411	3.0250	1.0173
2.2800	1.0780	2.5300	1.0124	2.7800	0.9421	3.0300	1.0251
2.2850	1.0827	2.5350	1.0008	2.7850	0.9447	3.0350	1.0323
2.2900	1.0853	2.5400	0.9894	2.7900	0.9488	3.0400	1.0386
2.2950	1.0856	2.5450	0.9784	2.7950	0.9542	3.0450	1.0438

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
3.0500	1.0479	3.3000	1.0075	3.5500	0.9620	3.8000	1.0159
3.0550	1.0508	3.3050	1.0002	3.5550	0.9636	3.8050	1.0207
3.0600	1.0523	3.3100	0.9930	3.5600	0.9662	3.8100	1.0249
3.0650	1.0524	3.3150	0.9860	3.5650	0.9697	3.8150	1.0285
3.0700	1.0513	3.3200	0.9794	3.5700	0.9740	3.8200	1.0314
3.0750	1.0487	3.3250	0.9735	3.5750	0.9789	3.8250	1.0334
3.0800	1.0450	3.3300	0.9683	3.5800	0.9845	3.8300	1.0346
3.0850	1.0401	3.3350	0.9640	3.5850	0.9904	3.8350	1.0348
3.0900	1.0342	3.3400	0.9607	3.5900	0.9966	3.8400	1.0341
3.0950	1.0275	3.3450	0.9585	3.5950	1.0029	3.8450	1.0326
3.1000	1.0201	3.3500	0.9575	3.6000	1.0090	3.8500	1.0302
3.1050	1.0123	3.3550	0.9576	3.6050	1.0150	3.8550	1.0271
3.1100	1.0042	3.3600	0.9588	3.6100	1.0205	3.8600	1.0233
3.1150	0.9961	3.3650	0.9612	3.6150	1.0255	3.8650	1.0189
3.1200	0.9881	3.3700	0.9646	3.6200	1.0298	3.8700	1.0140
3.1250	0.9806	3.3750	0.9689	3.6250	1.0333	3.8750	1.0088
3.1300	0.9737	3.3800	0.9741	3.6300	1.0359	3.8800	1.0034
3.1350	0.9675	3.3850	0.9800	3.6350	1.0376	3.8850	0.9980
3.1400	0.9622	3.3900	0.9864	3.6400	1.0384	3.8900	0.9927
3.1450	0.9580	3.3950	0.9931	3.6450	1.0381	3.8950	0.9876
3.1500	0.9550	3.4000	1.0000	3.6500	1.0368	3.9000	0.9829
3.1550	0.9532	3.4050	1.0069	3.6550	1.0346	3.9050	0.9787
3.1600	0.9527	3.4100	1.0136	3.6600	1.0315	3.9100	0.9750
3.1650	0.9535	3.4150	1.0200	3.6650	1.0276	3.9150	0.9721
3.1700	0.9556	3.4200	1.0258	3.6700	1.0230	3.9200	0.9700
3.1750	0.9588	3.4250	1.0309	3.6750	1.0179	3.9250	0.9686
3.1800	0.9632	3.4300	1.0352	3.6800	1.0123	3.9300	0.9681
3.1850	0.9685	3.4350	1.0385	3.6850	1.0065	3.9350	0.9685
3.1900	0.9747	3.4400	1.0409	3.6900	1.0005	3.9400	0.9698
3.1950	0.9815	3.4450	1.0422	3.6950	0.9946	3.9450	0.9718
3.2000	0.9888	3.4500	1.0424	3.7000	0.9889	3.9500	0.9746
3.2050	0.9964	3.4550	1.0415	3.7050	0.9835	3.9550	0.9781
3.2100	1.0041	3.4600	1.0395	3.7100	0.9786	3.9600	0.9821
3.2150	1.0117	3.4650	1.0365	3.7150	0.9743	3.9650	0.9866
3.2200	1.0189	3.4700	1.0326	3.7200	0.9707	3.9700	0.9915
3.2250	1.0257	3.4750	1.0279	3.7250	0.9680	3.9750	0.9966
3.2300	1.0317	3.4800	1.0225	3.7300	0.9661	3.9800	1.0018
3.2350	1.0370	3.4850	1.0165	3.7350	0.9651	3.9850	1.0069
3.2400	1.0412	3.4900	1.0102	3.7400	0.9651	3.9900	1.0118
3.2450	1.0443	3.4950	1.0036	3.7450	0.9661	3.9950	1.0164
3.2500	1.0463	3.5000	0.9971	3.7500	0.9679	4.0000	1.0206
3.2550	1.0471	3.5050	0.9906	3.7550	0.9706	4.0050	1.0242
3.2600	1.0467	3.5100	0.9845	3.7600	0.9741	4.0100	1.0272
3.2650	1.0450	3.5150	0.9788	3.7650	0.9783	4.0150	1.0295
3.2700	1.0422	3.5200	0.9737	3.7700	0.9831	4.0200	1.0310
3.2750	1.0384	3.5250	0.9694	3.7750	0.9883	4.0250	1.0316
3.2800	1.0335	3.5300	0.9660	3.7800	0.9938	4.0300	1.0315
3.2850	1.0279	3.5350	0.9634	3.7850	0.9995	4.0350	1.0306
3.2900	1.0215	3.5400	0.9619	3.7900	1.0051	4.0400	1.0288
3.2950	1.0147	3.5450	0.9615	3.7950	1.0106	4.0450	1.0263

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
4.0500	1.0232	4.3000	0.9770	4.5500	0.9945	4.8000	1.0223
4.0550	1.0195	4.3050	0.9751	4.5550	0.9984	4.8050	1.0219
4.0600	1.0153	4.3100	0.9739	4.5600	1.0024	4.8100	1.0208
4.0650	1.0107	4.3150	0.9734	4.5650	1.0063	4.8150	1.0192
4.0700	1.0059	4.3200	0.9736	4.5700	1.0100	4.8200	1.0172
4.0750	1.0010	4.3250	0.9745	4.5750	1.0135	4.8250	1.0147
4.0800	0.9960	4.3300	0.9761	4.5800	1.0166	4.8300	1.0118
4.0850	0.9913	4.3350	0.9784	4.5850	1.0192	4.8350	1.0086
4.0900	0.9867	4.3400	0.9812	4.5900	1.0214	4.8400	1.0053
4.0950	0.9826	4.3450	0.9845	4.5950	1.0230	4.8450	1.0018
4.1000	0.9790	4.3500	0.9882	4.6000	1.0239	4.8500	0.9983
4.1050	0.9760	4.3550	0.9923	4.6050	1.0243	4.8550	0.9949
4.1100	0.9736	4.3600	0.9965	4.6100	1.0240	4.8600	0.9916
4.1150	0.9720	4.3650	1.0008	4.6150	1.0232	4.8650	0.9886
4.1200	0.9711	4.3700	1.0051	4.6200	1.0217	4.8700	0.9859
4.1250	0.9710	4.3750	1.0093	4.6250	1.0196	4.8750	0.9836
4.1300	0.9717	4.3800	1.0132	4.6300	1.0171	4.8800	0.9817
4.1350	0.9731	4.3850	1.0167	4.6350	1.0141	4.8850	0.9804
4.1400	0.9753	4.3900	1.0198	4.6400	1.0108	4.8900	0.9796
4.1450	0.9781	4.3950	1.0224	4.6450	1.0072	4.8950	0.9793
4.1500	0.9815	4.4000	1.0244	4.6500	1.0035	4.9000	0.9796
4.1550	0.9854	4.4050	1.0257	4.6550	0.9997	4.9050	0.9804
4.1600	0.9897	4.4100	1.0264	4.6600	0.9959	4.9100	0.9818
4.1650	0.9942	4.4150	1.0264	4.6650	0.9923	4.9150	0.9836
4.1700	0.9989	4.4200	1.0257	4.6700	0.9889	4.9200	0.9859
4.1750	1.0036	4.4250	1.0244	4.6750	0.9858	4.9250	0.9886
4.1800	1.0082	4.4300	1.0224	4.6800	0.9831	4.9300	0.9915
4.1850	1.0126	4.4350	1.0198	4.6850	0.9809	4.9350	0.9947
4.1900	1.0167	4.4400	1.0168	4.6900	0.9792	4.9400	0.9980
4.1950	1.0203	4.4450	1.0133	4.6950	0.9781	4.9450	1.0014
4.2000	1.0233	4.4500	1.0095	4.7000	0.9776	4.9500	1.0047
4.2050	1.0258	4.4550	1.0055	4.7050	0.9776	4.9550	1.0079
4.2100	1.0276	4.4600	1.0014	4.7100	0.9783	4.9600	1.0109
4.2150	1.0286	4.4650	0.9973	4.7150	0.9796	4.9650	1.0136
4.2200	1.0289	4.4700	0.9932	4.7200	0.9814	4.9700	1.0159
4.2250	1.0285	4.4750	0.9894	4.7250	0.9837	4.9750	1.0178
4.2300	1.0273	4.4800	0.9859	4.7300	0.9864	4.9800	1.0192
4.2350	1.0254	4.4850	0.9828	4.7350	0.9895	4.9850	1.0202
4.2400	1.0229	4.4900	0.9802	4.7400	0.9928	4.9900	1.0206
4.2450	1.0198	4.4950	0.9781	4.7450	0.9964	4.9950	1.0205
4.2500	1.0162	4.5000	0.9766	4.7500	1.0000	5.0000	1.0198
4.2550	1.0122	4.5050	0.9758	4.7550	1.0037	5.0050	1.0187
4.2600	1.0079	4.5100	0.9756	4.7600	1.0072	5.0100	1.0170
4.2650	1.0034	4.5150	0.9761	4.7650	1.0105	5.0150	1.0149
4.2700	0.9989	4.5200	0.9772	4.7700	1.0136	5.0200	1.0125
4.2750	0.9945	4.5250	0.9789	4.7750	1.0163	5.0250	1.0097
4.2800	0.9902	4.5300	0.9812	4.7800	1.0185	5.0300	1.0067
4.2850	0.9862	4.5350	0.9840	4.7850	1.0203	5.0350	1.0036
4.2900	0.9826	4.5400	0.9872	4.7900	1.0216	5.0400	1.0004
4.2950	0.9795	4.5450	0.9907	4.7950	1.0222	5.0450	0.9971

the plotting; this subroutine interpolates additional points where they are needed, using a cubic interpolating polynomial.

The program was then modified so that it calculates the radar cross section, in both square meters and square feet, of a specific sphere at a specific frequency. Printouts of sample calculations using this program are shown as Figs. 2, 3, and 4. A listing of the program is given later in this report.

MATHEMATICAL FORMULAS

Kerr[1] gives the following equation (p. 451) for the radar cross section σ of a sphere of radius a:

$$\frac{\sigma}{\pi a^2} = \frac{1}{\rho^2} \left| \sum_{n=1}^{\infty} (-1)^n \ (2n + 1) \ (a_n^{\ s} - b_n^{\ s}) \right|^2 \qquad (1)$$

where $\rho = 2\pi a/\lambda$, λ is the wavelength, and the $a_n^{\ s}$ and $b_n^{\ s}$ are terms of a "multipole expansion" -- that is, these terms are proportional to the amplitudes of magnetic and electric multipoles induced in the sphere by the incident wave. When the sphere is perfectly conducting (Kerr, p. 452):

$$a_n^{\ s} = - \frac{j_n(\rho)}{h_n^{(2)}(\rho)} \qquad (2)$$

$$b_n^{\ s} = \frac{-[\rho \ j_n(\rho)]'}{[\rho \ h_n^{(2)}(\rho)]'} \qquad (3)$$

where the primes denote differentiation with respect to the argument. The functions j_n and $h_n^{(2)}$ are, respectively, the spherical Bessel function of the first kind, and the spherical Hankel function of the second kind.

A subroutine to evaluate j_n, j_n', and also the spherical Bessel function of the second kind and its derivative, y_n and y_n', was obtained from the NRL CDC-3800 program library.* No subroutine for evaluating the

*Library Catalog Identification C3-UCSD-BFFGH. Subroutine BFFGH was written by Frank Hagin of Texas Instruments, Dallas Texas.

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CALCULATION OF RADAR CROSS SECTION OF SPHERE
REFERENCE - KERR, ≠PROPAGATION OF SHORT RADIO WAVES≠
RAD LAB SERIES VOL, 13, P, 451, EQ, 29,

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INPUT DATA AND DERIVED QUANTITIES --

     RADIUS, METERS ..............................     1,524
     RADIUS, FEET ................................     5,000
     RADAR FREQUENCY, MEGAHERTZ ..................    138,60
     WAVELENGTH, METERS ..........................      2,16
     RADIUS/WAVELENGTH RATIO .....................    0,7046
     CIRCUMFERENCE/WAVELENGTH RATIO ..............    4,4270


OUTPUT DATA --

     NUMBER OF SERIES TERMS ADDED ................        12
     RADAR CROSS SECTION, NORMALIZED TO OPTICAL       0,9599
     RADAR CROSS SECTION, SQUARE METERS ..........    7,0038
     RADAR CROSS SECTION, SQUARE FEET ............   75,3884
```

Fig. 2 - Computer printout from Program SPHERE for radius 5 feet and frequency 138.6 MHz.

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CALCULATION OF RADAR CROSS SECTION OF SPHERE
REFERENCE - KERR, ≠PROPAGATION OF SHORT RADIO WAVES≠
RAD LAB SERIES VOL, 13, P, 451, EQ, 29,

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INPUT DATA AND DERIVED QUANTITIES --

     RADIUS, CENTIMETERS ........................      50,00
     RADIUS, FEET ...............................      1,640
     RADAR FREQUENCY, MEGAHERTZ .................    1000,00
     WAVELENGTH, CENTIMETERS ....................      29,98
     RADIUS/WAVELENGTH RATIO ....................     1,6678
     CIRCUMFERENCE/WAVELENGTH RATIO .............    10,4792


OUTPUT DATA --

     NUMBER OF SERIES TERMS ADDED ...............         20
     RADAR CROSS SECTION, NORMALIZED TO OPTICAL       0,9984
     RADAR CROSS SECTION, SQUARE METERS .........     0,7841
        SQUARE METERS, EXPONENT FORMAT ..........7,8411-001
     RADAR CROSS SECTION, SQUARE FEET ...........     8,4401
```

Fig. 3 - Computer printout from Program SPHERE for radius 0.5 meter and frequency 1000 MHz.

```
CALCULATION OF RADAR CROSS SECTION OF SPHERE
REFERENCE - KERR, ≠PROPAGATION OF SHORT RADIO WAVES≠
RAD LAB SERIES VOL, 13, P, 451, EQ, 29,

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INPUT DATA AND DERIVED QUANTITIES --

     RADIUS, CENTIMETERS ........................         6,00
     RADIUS, INCHES .............................         2,36
     RADAR FREQUENCY, MEGAHERTZ .................      1000,00
     WAVELENGTH, CENTIMETERS ....................        29,98
     RADIUS/WAVELENGTH RATIO ....................       0,2001
     CIRCUMFERENCE/WAVELENGTH RATIO .............       1,2575


OUTPUT DATA --

     NUMBER OF SERIES TERMS ADDED ...............            6
     RADAR CROSS SECTION, NORMALIZED TO OPTICAL         2,7464
     RADAR CROSS SECTION, SQUARE METERS .........       0,0311
        SQUARE METERS, EXPONENT FORMAT ..........3.1061-002
     RADAR CROSS SECTION, SQUARE FEET ...........       0,3343
        SQUARE FEET, EXPONENT FORMAT ............3.3434-001
```

Fig. 4 - Computer printout from Program SPHERE for radius 0.06 meter and frequency 1000 MHz.

spherical Hankel function was available. However, the problem was solved by the use of an expression relating the spherical Hankel function to the spherical Bessel functions. The necessary relation is given by Messiah (5):

$$h_k^{(2)} = j_k - iy_k$$

$$\therefore h_k^{(2)\prime} = j_k' - iy_k'$$

Since Subroutine BFFGH gives j_k, j_k', y_k, and y_k', it is a simple matter to obtain $h_k^{(2)}$ and $h_k^{(2)\prime}$. (To interpret the relations given by Messiah, it is necessary to note that his function n_k is the negative of y_k, and his functions $h_k^{(+)}$ and $h_k^{(-)}$ are equal to $ih_k^{(1)}$ and $-ih_k^{(2)}$, respectively.)

For very small values of ρ, the approximation*

$$\frac{\sigma}{\pi a^2} = 9\rho^4 = 1.4027\ (a/\lambda)^4 \times 10^4 \qquad (4)$$

can be used; this is the well known Rayleigh scattering law (for back scattering).

For very large values of ρ (or a/λ), the asymptotic result is

$$\frac{\sigma}{\pi a^2} \approx 1$$

As will be discussed in some detail, this approximation is valid to 4 significant figures for $a/\lambda > 44$ (and possibly for somewhat smaller values).

FORTRAN PROGRAM

The program written to compute the cross section is simply a Fortran algorithm to evaluate Eq. (1). The summation of multipole terms (which are complex numbers) is carried to the point to which the fractional change in σ

---

*Reference 1, page 452.

due to the last term added is less than 10^{-6}. No exact analysis of the accuracy obtained by this procedure was made; however, a check on accuracy is afforded by the knowledge that as a/λ becomes large, the envelope of the oscillations of σ asymptotically approaches πa^2. The numerical results obtained for large a/λ agree with this prediction. To test this, the program was run with the value of a/λ doubled for each successive calculation. The results are shown in Table 2. The initial value of a/λ was .0850 and this was doubled until the value 21.76 was reached, at which time the run terminated. An additional run was then made for the values $a/\lambda = 44$ and $a/\lambda = 88$. The second column of the table gives the number of series terms summed before the above-stated criterion was met (fractional change in $\sigma < 10^{-6}$).

Table 2

Results for Large Values of a/λ

a/λ	No. of Terms Summed	$\sigma/\pi a^2$
.0850	5	0.6800
.1700	6	3.6208
.3400	7	1.5845
.6800	11	0.7001
1.3600	17	1.1286
2.7200	28	1.0156
5.4400	47	0.9929
10.8800	83	0.9970
21.7600	158	0.9997
44.0000	303	1.0000
88.0000	586	1.0000

It is seen that for $a/\lambda \geqq 44$, the values of $\sigma/\pi a^2$ differ from 1 by less than 5×10^{-6}. This suggests that the calculation is accurate to at least 4 significant figures for values of a/λ up to at least 88. To make sure that the values 1.0000 were not obtained "by accident" at $a/\lambda = 44$, the program was run with values $a/\lambda = 44.04$ and 44.045. These numbers were chosen to insure that if a/λ just happened to result in $\sigma/\pi a^2 = 1.0000$ when the value is still actually oscillating appreciably, the oscillation would be revealed. The results, printed out to additional decimal places, were $\sigma/\pi a^2 = 0.999982$ and 0.999981, respectively. It thus seems reasonable to conclude that the calculation does have at least 4-significant-figure accuracy.

Since for large values of a/λ the number of terms summed almost doubles with each doubling of a/λ, a point would eventually be reached at which the accuracy would be reduced by computer round-off error. However, in the CDC-3800 this event occurs at a value of a/λ well above the point at which the approximation $\sigma/\pi a^2 \approx 1$ can be assumed to 4 significant figures. It is therefore recommended that for values of $a/\lambda > 44$, the optical approximation $\sigma/\pi a^2 = 1.000$ be used instead of making an actual computer calculation.

The time required for the calculation with the CDC-3800 is approximately 58 milliseconds per series term, not including compilation time and input-output operations. Therefore calculation of single values of σ for the usual range of values of a/λ requires only a few seconds at most.

A listing of the Fortran program that has output of the type shown in Figs. 2-4 follows. The input to this program is a data card or cards (one for each combination of sphere size and frequency to be calculated). Any number of data cards can be stacked at the appropriate place in the deck, and followed by an end-of-file card. The program will then read each card, do the appropriate computations, and print out the results. The data card format is:

Card Columns	Quantity	Format Specification
1-10	Radius of sphere	F10.
11-20	Radar frequency	F10.
30	Radius units designator	I1

The radius of the sphere can be given in either meters or feet. If it is given in meters, Column 30 is left blank. If it is given in feet, a 1 is punched in Column 30. The radar frequency is given in megahertz. Both the radius and the frequency can be punched as floating-point numbers anywhere within the specified fields; a decimal point must be included unless the numbers are integers and are right-adjusted in the field.

As mentioned in the introduction, the computer program that follows was written before the paper of Adler and Johnson (3) was called to the author's attention. They describe a method of computing the spherical Bessel and Hankel functions based on a recursion relation. Use of this method instead of Subroutine BFFGH would have been somewhat more direct and possibly would reduce computing time; however, since the program as it stands works well and runs without excessive use of computer time, it was decided not to change it.

```
                                                              03/06/72
      PROGRAM SPHERE
C
C     COMPUTES RADAR CROSS SECTION OF A PERFECTLY CONDUCTING SPHERE.
C     MATHEMATICS GIVEN BY KERR, *PROPAGATION OF SHORT RADIO WAVES*
C     RAD LAB SERIES VOL. 13 (MCGRAW-HILL, 1950), P. 451, EQ. 29.
C     PROGRAM WRITTEN BY L. V. BLAKE, NRL CODE 5370. THIS VERSION 3/1/72
C
C     COMPUTES AS MANY CASES (SPHERE SIZES, FREQUENCIES) AS THERE ARE
C     DATA CARDS.  RADIUS OF SPHERE, COLS. 1-10. FREQUENCY MEGAHERTZ,
C     COLS. 11-20.  IF RADIUS IS GIVEN IN FEET, PUNCH A 1 IN COL. 30.
C     IF RADIUS IS GIVEN IN METERS, NO PUNCH (OR ZERO PUNCH) IN COL. 30.
C
      TYPE COMPLEX HN2,DHN2,ANS,BNS,TERM,SUM
      DATA (PI=3.1415926536), (M=0)
  500 READ 30, RADIUS, FRQNCY, IUNITS
      IF (EOF,60) 88,89
   88 STOP
   89 IF (IUNITS.EQ.1) 31,32
   31 RADFT=RADIUS
      RADIUS=RADIUS*.3048
      GO TO 33
   32 RADFT=RADIUS/.3048
   33 WVLGTH=299.7925/FRQNCY
      RATIO= RADIUS/WVLGTH
      RHO = 2.*PI*RATIO
      N = 0
      QSQ = 1.
      IKQ = 1
      Y = 1.
      N = 0
      SUMLAST = 0.
      ISIGN = 1
      SUM = CMPLX(0.,0.)
    1 N = N+1
      ISIGN = ISIGN*(-1)
      CALL BFFGH(RHO,N,BJ,DBJ,DDBJ,BY,DBY,DDBY,QSQ,IQK,Y)
      HN2 = CMPLX(BJ,-BY)
      DHN2 = CMPLX(DBJ,-DBY)
      ANS = -BJ/HN2
      BNS = -(RHO*DBJ+BJ)/(RHO*DHN2+HN2)
      TERM = ISIGN*(2*N+1)*(ANS-BNS)
      SUM = SUM+TERM
      ABSUM2 = (CABS(SUM))**2
      DIFFR=ABS ((ABSUM2-SUMLAST)/ABSUM2)
      IF (DIFFR .LT. 1.E-6) GO TO 9
      SUMLAST = ABSUM2
      GO TO 1
    9 SIGNML   = ABSUM2/(RHO*RHO)
      SIGMA=SIGNML*PI*RADIUS*RADIUS
      SIGFT=SIGMA*10.76391
      IF (M.EQ.0) 80,81
   80 M=1
      GO TO 82
   81 PRINT 998
   82 PRINT 992
      PRINT 99
```

```
      PRINT 990
      PRINT 991
      PRINT 999
      PRINT 100
      IF (RADIUS .GE. 1.) 40,41
   40 PRINT 101, RADIUS
   43 PRINT 102, RADFT
      GO TO 42
   41 RADCM=RADIUS*100.
      PRINT 1011, RADCM
      IF (RADFT .GE. 1.) 43,44
   44 RADIN=RADFT*12.
      PRINT 1012, RADIN
   42 PRINT 103,FRQNCY
      IF (WVLGTH.GE.1.) 50,51
   50 PRINT 104, WVLGTH
      GO TO 52
   51 WVLGHH=WVLGTH*100.
      PRINT 105,WVLGHH
   52 PRINT 106, RATIO
      PRINT 107, RHO
      PRINT 108
      PRINT 1080, N
      PRINT 109, SIGNML
      PRINT 110, SIGMA
      IF (SIGMA .LT. 1. .OR. SIGMA .GT. 99999.9999) PRINT 112, SIGMA
      PRINT 111, SIGFT
      IF (SIGFT .LT. 1. .OR. SIGFT .GT. 99999.9999) PRINT 113, SIGFT
      GO TO 500
   30 FORMAT(2F10.2,9X,I1)
  998 FORMAT(1H1)
  992 FORMAT(//////)
   99 FORMAT(15X,*CALCULATION OF RADAR CROSS SECTION OF SPHERE*  )
  990 FORMAT(15X,*REFERENCE - KERR, ≠PROPAGATION OF SHORT RADIO WAVES≠*)
  991 FORMAT(15X,*RAD LAB SERIES VOL. 13, P. 451, EQ. 29,    *//)
  999 FORMAT(15X,*---------------------------------------------*///)
  100 FORMAT(15X,*INPUT DATA AND DERIVED QUANTITIES --*/)
  101 FORMAT(20X,*RADIUS, METERS ...............................*,F10.3  )
 1011 FORMAT(20X,*RADIUS, CENTIMETERS ..........................*,F10.2  )
 1012 FORMAT(20X,*RADIUS, INCHES ...............................*,F10.2  )
  102 FORMAT(20X,*RADIUS, FEET .................................*,F10.3  )
  103 FORMAT(20X,*RADAR FREQUENCY, MEGAHERTZ ...................*,F10.2  )
  104 FORMAT(20X,*WAVELENGTH, METERS ...........................*,F10.2  )
  105 FORMAT(20X,*WAVELENGTH, CENTIMETERS ......................*,F10.2  )
  106 FORMAT(20X,*RADIUS/WAVELENGTH RATIO ......................*,F10.4  )
  107 FORMAT(20X,*CIRCUMFERENCE/WAVELENGTH RATIO .............*F10.4///)
  108 FORMAT(15X,*OUTPUT DATA --*/)
 1080 FORMAT(20X,*NUMBER OF SERIES TERMS ADDED ................*,5X,I5  )
  109 FORMAT(20X,*RADAR CROSS SECTION, NORMALIZED TO OPTICAL  *,F10.4  )
  110 FORMAT(20X,*RADAR CROSS SECTION, SQUARE METERS ..........*,F10.4  )
  112 FORMAT(23X,*SQUARE METERS, EXPONENT FORMAT ............*,E10.4    )
  111 FORMAT(20X,*RADAR CROSS SECTION, SQUARE FEET ............*,F10.4  )
  113 FORMAT(23X,*SQUARE FEET, EXPONENT FORMAT ..............*,E10.4    )
      END
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      SUBROUTINE BFFGH (Z, L, BA, BJP, BJPP, BB, BYP, BYPP,QSQ,IKQ,Y)
C     C3 UCSD BFFGH
      DIMENSION BJ(5), BJL(5), BY(5), BYL(5),R(3)
      YSAVE = Y
      IF (Y - 1,) 102, 102, 101
  101 Y = 1,
  102 FL = L
      L1=L+1
      ZB = Z
      ZE = 0,
      FL1 = FL + 1,
      ZSQ = Z * Z
      FLR = SQRTF (FL * FL1)
      SINE = SINF(Z)
      COSE = COSF(Z)
      IF (IKQ - 1) 104,104,108
  100 IF (Z - FLR) 800,800,900
  104 IF (Z-1,) 300,200,200
  108 IF (QSQ) 110, 110, 104
  110 IF (Z - 1,) 500, 400, 400
  200 BJ(1) = SINE / ZB
      SIGN = 1.
      ER = -Y
      BY(1) = ER * COSE
  210 BY(1) = BY(1) / ZB
      ER = BJ(1) / ZB
      IF (SIGN) 211,212,212
  211 ER = -ER
  212 BJ(2) =(BY(1)/Y  + ER)/ Y
      BY(2) = (BY(1)/Z - Y*BJ(1))*Y
      FLX = 2,
  221 TEMP =(2, *FLX -1,)/Z
      BJ(3)=(TEMP * BJ(2) - BJ(1)/Y)/Y
      IF (SIGN) 223,224,224
  223 BJ(3) = -BJ(3)
  224 ER    =     TEMP * BY(2)
      BY(3) =   Y * BY(1)
      IF (SIGN) 225,226,226
  225 BY(3) = -BY(3)
  226 BY(3) =((ER - BY(3)))* Y
      IF (FLX -FL1) 222,228,228
  228 IF (SIGN) 430,430,100
  222 BJ(1) = BJ(2)
      BY(1) = BY(2)
      BJ(2) = BJ(3)
      BY(2) = BY(3)
      FLX = FLX + 1,
      GO TO 221
  300 LX = L - 1
      SEXIT = 1,
  301 DO 320 I = 1, 3
  301 SUM2 = 1,
      SUM1 = 1.
      TERM1 = 1,
      TERM2 = 1,
      FM = 1,
```

```
      TL = 2 * LX
  302 TM = 2. * FM
      TERM1 =-TERM1 * ZSQ / (TM * (TL + TM + 1.)) * SEXIT
      TERM2 = TERM2 * ZSQ / (TM * (TL + TM + 1.)) * SEXIT
      OLD1 = SUM1
      OLD2 = SUM2
      SUM1 = SUM1 + TERM1
      SUM2 = SUM2 + TERM2
      DIFF = ABSF((OLD1 - SUM1) / SUM1) + ABSF((OLD2 - SUM2) / SUM2)
      IF (DIFF - .000001) 306, 306, 304
  304 FM =FM + 1.
      GO TO 302
  306 IX= 2 * LX- 1
      ASM = -1.
      IF (LX) 310, 310, 307
  307 DO 308 J = 1, IX, 2
      AJ = J
  308 ASM = ASM * AJ
  310 ZYL = (Z / Y) ** LX
      ZYL1= ZYL * (Z / Y)
      IF (LX) 312, 312, 314
  312 ASM1 = 1.
      GO TO 316
  314 ALX = LX
      ASM1 = -ASM * (2. * ALX + 1.)
  316 BJ(I) = ZYL * SUM1 / ASM1
      BY(I) = ASM * SUM2 / ZYL1
  320 LX = LX + 1
      IF (SEXIT) 430,430,900
  400 SIGN = -1.
      ER = EXPF (ZB)
      ERL = 1. / ER
      SNH = .5 * (ER - ERL)
      CSH = .5 * (ER + ERL)
      BJ(1) = SNH / ZB
      BY(1) = Y * CSH
      GO TO 210
  500 LX = L - 1
      SEXIT = -1.
      GO TO 391
  430 KNT = 2
      BY(1) = Y/ZB
      BY(2) = Y*Y*(1.+ZB) / (ZB*ZB)
  583 BY(3) = (2.*FLOATF(KNT) -1.)*Y*BY(2) / ZB + Y*Y*BY(1)
      IF(KNT-L-1) 582,581,581
  582 BY(1) = BY(2)
      BY(2) = BY(3)
      KNT = KNT + 1
      GO TO 583
  581 EXZ = EXPF(-ZB)
      DO 584 N = 1,3
  584 BY(N) = BY(N) * EXZ
      IF (ZB-1.) 600,580,580
  580 IF (Z-4.*FLR) 700,600,600
  600 ER = 2.* FL  + 1.
      BJP = (FL * BJ(1) / Y + FL1 * BJ(3) * Y) / ER
```

```
      BYP =-(FL * BY(1) * Y + FL1 * BY(3) / Y) / ER
      LOC=1
      IF (L.EQ.0) GO TO 1100
      BJPP = 2. * BJP / Z  - (1. + FL * FL1 / ZSQ) * BJ(2)
      BYPP = 2. * BYP / Z  - (1. + FL * FL1 / ZSQ) * BY(2)
      GO TO 950
  800 A  = 0.1
      IZE = 0
      B  = 0.35
      L1 = L+1
      FN1 = L
      IDX = -1
      FN2 = Z - 0.5 + SQRTF(30.0 * B * Z)
      ZE = 0.
      U1 = 2. * Z/(2.* FN1 + 1.)
      U2 = 2. * Z/(2.* FN2 + 1.)
      M1 = FN1 + 30. * (A + B*U1*(2.-U1*U1)/(2. *(1. - U1 * U1)))
      M2 = FN2 + 30. * (A + B*U2*(2.-U2*U2)/(2. *(1. - U2 * U2)))
      AL = L
      IF (FN2-AL) 801,801,802
  801 AM = M1 + 1
      M = M1
      GO TO 803
  802 IF (M1 - M2)  801,801,805
  805 AM = M2 +1
      M = M2
  803 R(3) =  Z/(2. * AM  + 3.)
      N = M
  808 AN = N
      R(2) =  Z/(2. * AN + 3. - Z * R(3))
      N = N -1
      IF (L - N) 806,806,807
  806 R(3) = R(2)
      GO TO 808
  807 BJ(3) = R(2)
      BJ(2) = 1.
      AL = 2*L +1
      BJ(1) = AL/Z - R(2)
  809 LA = -L
      LA1=-L-1
      ALPHA = Z*Z*(BJ(2)*BY(1)*Y**LA-BJ(1)*BY(2)*Y**LA1)
      NA = L-1
      DO 812 N=1,3
      BJ(N)=(1./(Y**NA*ALPHA)) *BJ(N)
  812 NA=NA+1
      GO TO 900
  700 A = 0.1
      B = .35
      FN = -.5 + SQRTF(30. * B * ZB)
      IF (FN-FL) 701,702,702
  701 FN = FL
  702 U = 2. * ZB /(2. * FN + 1.)
      M = FN + 30. * (A + B * U) + 1.
      R(3) =  ZB/ 2. * FLOATF(M) + 1.
      M  = M - 1
  705 AN = M
```

```
      R(2) = ZB/(2. * AN + 1. + ZB * R(3))
      M = M - 1
      IF (M - L -1) 703,704,704
  704 R(3) = R(2)
      G0 T0 705
  703 BJ(3) = R(2)
      BJ(2) = 1.
      BJ(1) =(2.*FL + 1.)/ZB  *R(2)
      ALPHA=ZB*ZB*(BJ(2)*BY(1)/Y**L + BJ(1)*BY(2)/Y**L1)
      NA=FL-1.
      D0 712 N=1,3
      BJ(N)= (1./(Y**NA*ALPHA)) * BJ(N)
  712 NA=NA+1
      G0 T0 600
  900 ER = 1. / (2. * FL + 1.)
      BJP = ER * (FL * BJ(1) / Y - FL1 * BJ(3) * Y)
      BYP = ER * (FL * BY(1) * Y - FL1 * BY(3) / Y)
      L0C=2
      IF (L.EQ.0) G0 T0 1100
      BJPP = (FL * FL1 / ZSQ - 1.) * BJ(2) - 2. * BJP / Z
      BYPP = (FL * FL1 / ZSQ - 1.) * BY(2) - 2. * BYP / Z
  950 BA = BJ(2)
      BB = BY(2)
 1000 Y = YSAVE
      RETURN
 1100 IF (IKQ.LE.1)1120,1110
 1110 IF (QSQ.LT.0)1150,1120
 1120 BA=SIN(Z)/Z
      BB=-C0S(Z)/Z
      IF(L0C-1)1130,1130,1140
 1130 BJPP=2.*BJP/Z-(1.+FL*FL1/ZSQ)*BA
      BYPP=2.*BYP/Z-(1.+FL*FL1/ZSQ)*BB
      RETURN
 1140 BJPP=(FL*FL1/ZSQ-1.)*BA-2.*BJP/Z
      BYPP=(FL*FL1/ZSQ-1.)*BB-2.*BYP/Z
      RETURN
 1150 BA=SINH(Z)/Z
      PI=3.1415926535
      BB=EXP(-Z)*PI/(2.*Z)
      IF (L0C-1)1130,1130,1140
      END


      FUNCTI0N SINH(Z)
      SINH=(EXP(Z)-EXP(-Z))/2.
      END
```

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